PACKAGE SURVIVAL PROBABILITY CONSIDERATION WITH APPLICATION TO AIRDROP OF MILITARY SUPPLIES.

Mohammed Rasjid Tirtakusuma

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

PACKAGE SURVIVAL PROBABILITY CONSIDERATION
WITH APPLICATION TO
AIRDROP OF MILITARY SUPPLIES

by

M. Rasjid Tirtakusuma. drs

March 1978

Thesis Advisor: J.B. Tysver

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T183650

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Package Survival Probability Consideration with Application to Airdrop of Military Supplies		5. TYPE OF REPORT & PERIOD COVERED Master's Thesis; March 1978 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) M. Rasjid Tirtakusuma. drs		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		12. REPORT DATE March 1978 13. NUMBER OF PAGES 67
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Limited Distribution.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Military Airdrop
Survival Probability of Packages

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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S/N 0102-014-6601 |
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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

(20. ABSTRACT Continued)

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by

M. Rasjid Tirtakusuma. drs
Major, Indonesian Air Force

Submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL

March 1978

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I. INTRODUCTION

Indonesia is not a continental country, it consists of five big islands, i.e., Java, Sumatra, Kalimantan, Sumatra, Sulawesi and West Irian, besides thousands of small islands, scattered between Asia and Australia. It spans approximately 3000 miles from west to east and 1500 miles from south to north.

Since its independence in 1945, Indonesia has experienced several uprisings which threatened its nation's integrity and this must be tackled in the shortest possible time in the form of military operations. More often than not, the troops involved must be supplied from other islands, particularly from Java as a supply center. Experience has shown that air delivery supply is the most preferred way to supply the troops and the supplies must be dropped from the aircraft because of lack of airfields in the trouble spots. Particularly if the trouble spots were jungle areas where most of the islands are.

The problem of packaging military supplies so as to increase the probability of survival of the packages is of greater interest nowadays. There are many situations in which packages of items are lost or damaged because of handling or packaging of the items and this will lead to decreasing probability of survival of the packages.

In packaging problems there are some elements of interest:

1. There are m items to be packaged; these may be either the total number of items produced or the number to be packaged for a single destination.
2. There are k items in each package.
3. A package which contains k items has a survival probability p_k .
4. The desired number of items surviving is y^* .

A more extensive discussion of the general packaging problem is presented in reference 1.

In military application, the survival probability of packages becomes of great concern. Listed below are a few military platforms (packages) carrying weapons (packaged items).

Platforms	Weapon
Bomber aircraft	Bombs or missiles
Transport aircraft	Troops or supplies
Fighter aircraft	Cannons or rockets
Submarine	Torpedos, missiles
Aircraft carrier	Aircraft

For each of these platforms, an appropriate package size must be selected so as to increase the probability that a desired number of items (y^*) will survive.

An application to airdrop of supplies will be developed in Section III. Military air supply has played an important role since WW II, with its application becoming increasingly

necessary and important, particularly for a country like Indonesia, which consists of many islands, on some of which the roads are in very poor condition or do not exist. In a situation like this, when a military operation occurs in some area, airdrop of supplies is essential. There are two types of aircraft available for airdrop supply in Indonesia, C-130B Hercules and C-47 Dakota, with payloads 20,000 kg and 4,000 kg respectively. There are two different packaging concepts concerning these aircraft for dropping items;

1. The aircraft as a conveyer. This means that the aircraft carries several containers as packages. Each container contains k items, and the capacity of the aircraft is the m items.
2. The aircraft itself is a package which carries k items.

An item can have various values of weight. 20 kg is chosen for an item weight since it can be carried by hand. A C-130B which has a payload of 20,000 kg thus can carry 1000 items and C-47 with payload 4000 kg can carry 200 items.

Possible package sizes (k items) will be restricted to numbers such that:

$$n_k = m/k \quad \text{is an integer and} \quad m = n_k \cdot k \ ,$$

where n_k is the number of packages, each of size k. No partially filled packages will be allowed.

In this thesis, emphasis is placed on the effects of varying the number of items in a package on the probability that at least a desired number of items (y*) survive, at various individual package survival probabilities (p_k) .

II. BASIC MODEL

A. BASIC CHARACTERISTICS

There are some basic characteristics which are common to all the applications mentioned in Section I in order to conform with the basic model, i.e.:

1. There is a finite number of items (m) to be packaged.
2. The number of items (k) to be placed in each package (package size) is to be selected.
3. There is a probability (p_k) that a package containing k items will survive.

There is a clear distinction between package survival and item survival. The C-130B aircraft can survive even though some or all of its supplies dropped do not.

B. BINOMIAL MODEL

To comply with the basic assumption concerning the independence of random variables in the Binomial Model, in a statistical sense, we have to draw assumptions concerning the packages:

1. Each package containing k items has the same probability of survival p_k .
2. The survival or failure to survive of any package does not affect the probability of survival of other packages.

Let X denote the number of packages that will survive, then X is a random variable which can take any values from 0 to n_k. It is clear that X is binomially distributed. If p is the survival probability of the package, then the probability of failure to survive is $(1-p)$. The probability that X will take on a certain integer value between 0 and n_k will be:

$$P(X = x) = \binom{n_k}{x} p^x (1-p)^{n_k - x} ,$$

where

$$\binom{n_k}{x} = \frac{n_k!}{x!\ (n_k - x)} .$$

The values of $P(X = x)$ with a certain n_k and p, can be found using Binomial Tables [Ref. 2] or can be computed with a programmable calculator (Texas Instruments TI 58).

Given the number of packages n_k and their survival probability p_k, the number of packages which can be expected to survive is:

$$\mu_k = n_k \cdot p_k \qquad (2)$$

The surviving number of packages must be an integer, but not the expected value.

C. THE LEAST NUMBER OF ITEMS TO SURVIVE ($P(Y \geq y)$)

In the previous subsection we had X as the number of surviving packages. If y is the number of items in those surviving packages, with k items in each package, then:

$$Y = k \cdot X \tag{3}$$

The expected number of surviving packaged items is:

$$\mu_y = k \cdot \mu_x \tag{4}$$

Following (2):

$$\mu_y = k \cdot n_k \cdot p_k$$

$$= m \cdot p_k$$

If all package sizes have the same survival probability $p_k = p$, then $\mu_y = m \cdot p$ and the consideration of survival probabilities would appear to be unimportant in determining the package size. The package sizes would then be determined by other considerations such as cost, control, convenience, etc. We'll see in the following analysis that survival probabilities do have a bearing on the selection of package sizes.

If the least number of items required to survive is y^* (y with an asterisk), then the probability that the number

of surviving items Y will be greater than or equal to y^* will be written: $P(Y \geq y^*)$. $P(Y \geq y^*)$ can be determined from the expression:

$$P(Y \geq y^*) = P(X \geq x^*)$$

$$= \sum_{X=x^*}^{n_k} P(X = x)$$

so that:

$$P(Y \geq y^*) = \sum_{x=x^*}^{n_k} \binom{n_k}{x} p_k^{\,x} (1 - p_k)^{n_k - x} \qquad (5)$$

where x is the number of packages that will survive and x^* is the largest integer which is less than or equal to y^*/k (At least enough packages (x^*) must survive so that y^* items survive).

The values of $P(Y \geq y^*)$ for some arbitrary values of y^* are presented in Tables 1 through 10 and the graphs are in Figures 1 through 10.

TABLE 1

Values of $P(Y \geq y^*)$ for Airdrop Using C-130B with $y^* = 1000$ Items

k	n	x*	p								
			.1	.2	.3	.4	.5	.6	.7	.8	.9
1,000	1	1	.1	.2	.3	.4	.5	.6	.7	.8	.9
500	2	2	.01	.04	.09	.16	.25	.36	.49	.64	.81
250	4	4	.0001	.0016	.0081	.0256	.0625	.1296	.2401	.4096	.6561
200	5	5	0.0000	.0003	.0024	.0102	.0313	.0778	.1681	.3277	.5905
125	8	8		0.0000	.0001	.0007	.0039	.0168	.0576	.1678	.4305
100	10	10			0.0000	.0001	.0010	.0060	.0282	.1074	.3487
50	20	20						0.0000	.0008	.0115	.1216
40	25	25							0.0000	.0038	.0718
25	40	40								.0001	.0052
20	50	50								0.0001	.0002
10	100	100									
5	200	200									

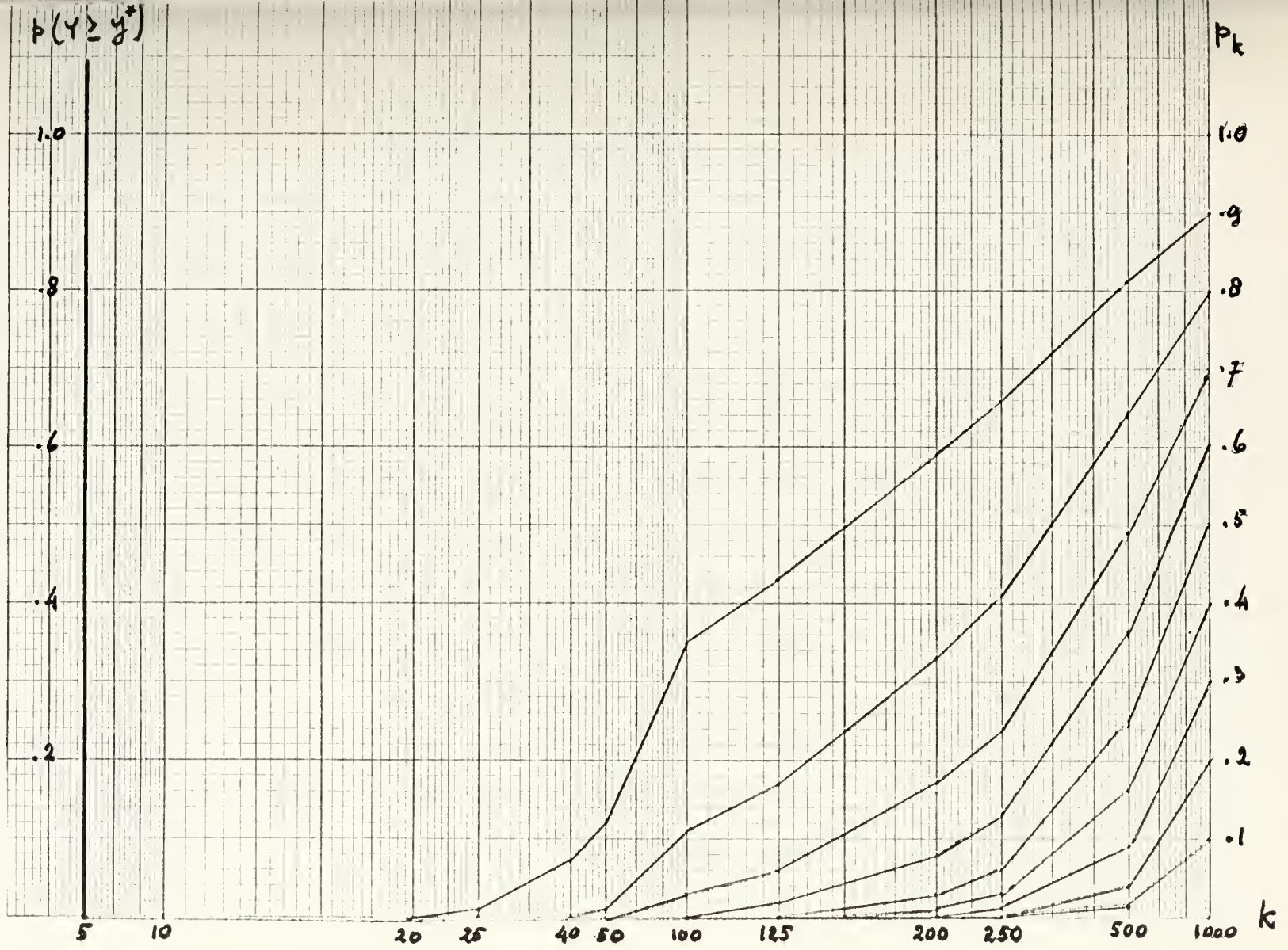


FIGURE 1. Graph of $P(Y \geq y^*)$ for Airdrop using C-130B with $y^* = 1000$ items

TABLE 2

Values of $P(Y \geq y^*)$ for Airdrop Using C-130B with $y^* = 500$ Items

k	n	x*	p								
			.1	.2	.3	.4	.5	.6	.7	.8	.9
1,000	1	1	.1	.2	.3	.4	.5	.6	.7	.8	.9
500	2	1	.19	.36	.51	.64	.75	.84	.91	.96	.99
250	4	2	.0523	.1808	.3483	.5248	.6875	.8208	.9163	.9728	.9963
200	5	3	.0086	.0579	.1631	.3174	.5000	.6826	.8369	.9421	.9996
125	8	4	.0050	.0563	.1941	.4059	.6367	.8263	.9420	.9896	1.0000
100	10	5	.0016	.0328	.1503	.3669	.6230	.8338	.9257	.9936	
50	20	10		.0026	.0480	.2447	.5881	.8725	.9829	.9994	
40	25	13		.0004	.0175	.1538	.5000	.8462	.9825	1.0000	
25	40	20			.0063	.1298	.5627	.9256	.9825		
20	50	25			.0024	.0978	.5561	.9427	.9976		
10	100	50			0.0000	.0271	.5398	.9832	.9991		
5	200	100				.0026	.5282	.9983	1.0000		

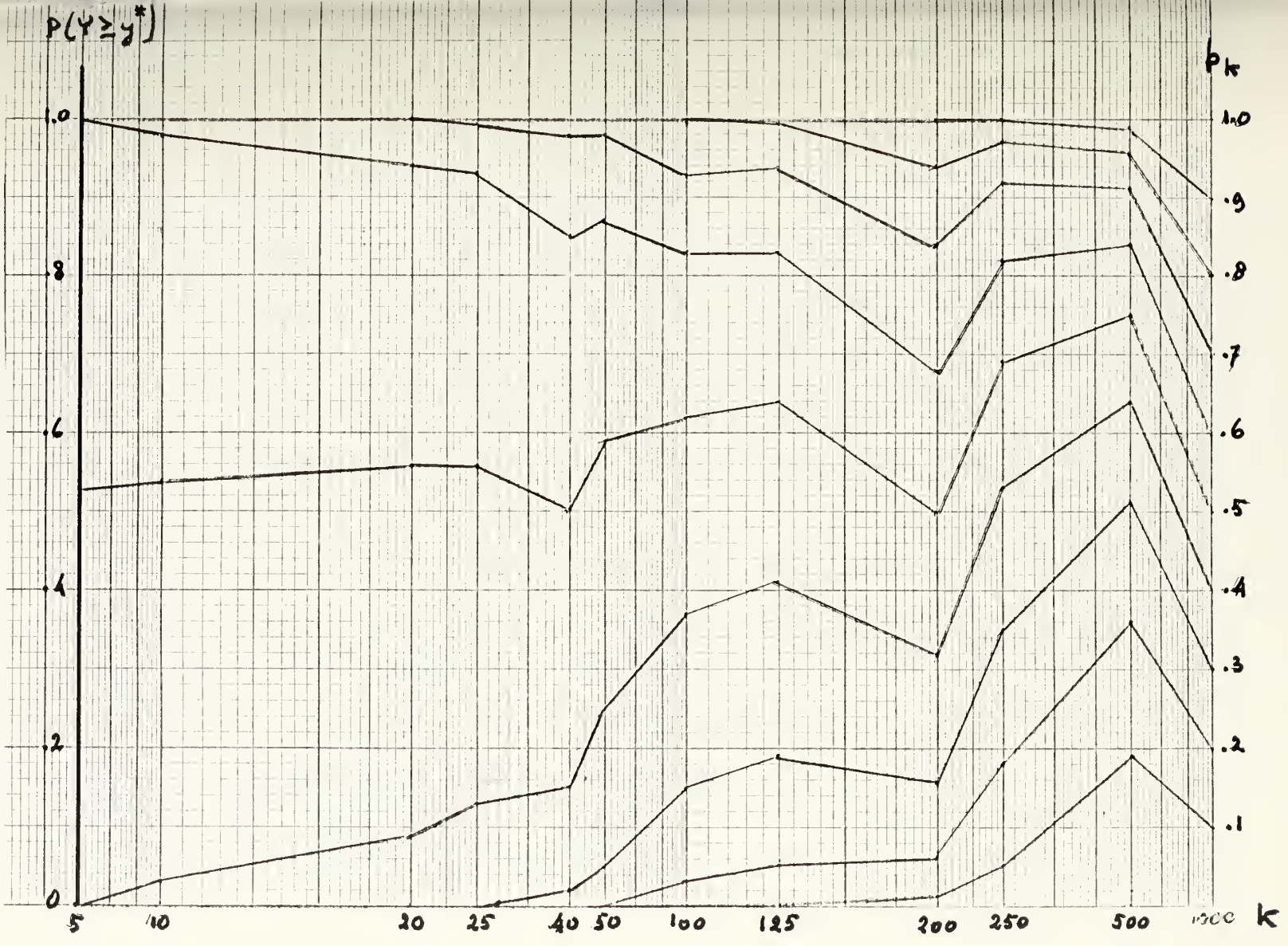


FIGURE 2. Graph of $P(Y \geq y^*)$ for Airdrop using C-130B with $y^* = 500$ items

TABLE 3

Values of $P(Y \geq y^*)$ for Airdrop Using C-130B with $y^* = 250$ Items

k	n	x*	p									
			.1	.2	.25	.3	.4	.5	.6	.7	.8	.9
1,000	1	1	.1	.2	.25	.3	.4	.5	.6	.7	.8	.9
500	2	1	.19	.36	.4375	.51	.64	.75	.84	.91	.96	.99
250	4	1	.3439	.5904	.6836	.7599	.8704	.9375	.9744	.9919	.9984	.9999
200	5	2	.0815	.2627	.3672	.4718	.6630	.8125	.9130	.9692	.9932	1.0000
125	8	2	.1869	.4967	.6329	.7447	.8930	.9648	.9915	.9987	.9999	
100	10	3	.0702	.3222	.4744	.6172	.8327	.9453	.9877	.9984	1.0000	
50	20	5	.0532	.3704	.5852	.7625	.9490	.9941	.9997	1.0000		
40	25	7	.0095	.2200	.4389	.6593	.9264	.9926	1.0000			
25	40	10	.0051	.2682	.5605	.8041	.9844	.9997				
20	50	13	.0010	.1860	.4890	.7771	.9868	1.0000				
10	100	25	0.0000	.1314	.5383	.8864	1.0000					
5	200	50		.0494	.5271	.9494						

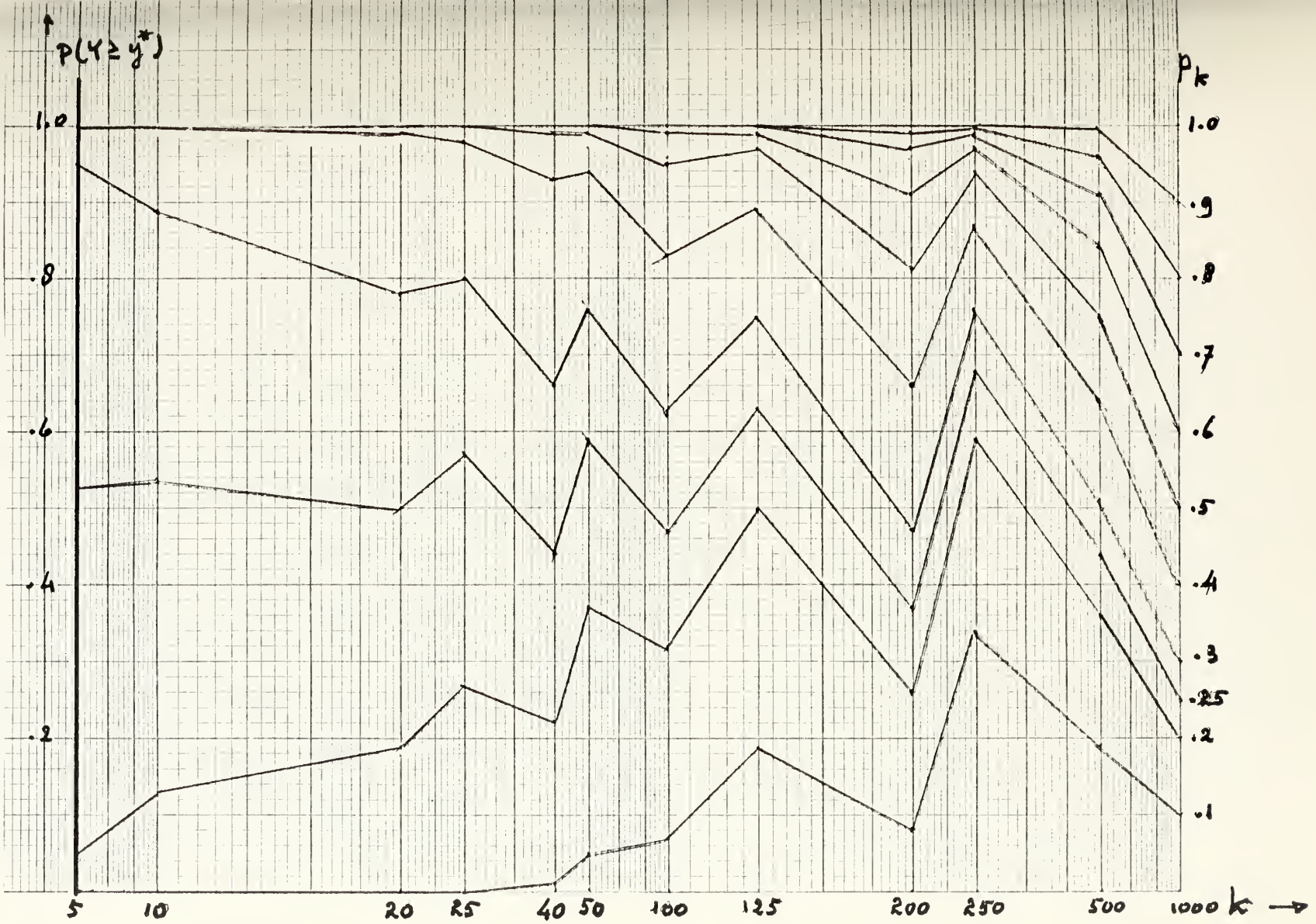


FIGURE 3. Graph of $P(Y \geq y^*)$ for Airdrop using C-130B with $y^* = 250$ items

TABLE 4

Values of $P(Y \geq y^*)$ for Airdrop Using C-130B with $y^* = 125$ Items

k	n	x*	p									
			.1	.125	.2	.3	.4	.5	.6	.7	.8	.9
1,000	1	1	.1	.1250	.2	.3	.4	.5	.6	.7	.8	.9
500	2	1	.19	.2344	.36	.51	.64	.75	.84	.91	.96	.99
250	4	1	.3439	.4138	.5904	.7599	.8704	.9375	.9744	.9919	.9984	.9999
200	5	1	.4095	.4871	.6723	.8319	.9222	.9688	.9887	.9975	1.0000	1.0000
125	8	1	.5695	.6564	.8322	.9424	.9832	.9961	.9993	1.0000		
100	10	2	.2639	.3611	.6242	.8507	.9536	.9893	1.0000			
50	20	3	.3231	.4647	.7939	.9645	.9964	1.0000				
40	25	4	.2364	.3824	.7660	.9668	1.0000					
25	40	5	.3710	.5713	.9241	.9974						
20	50	7	.2298	.4363	.8966	.9975						
10	100	13	.1982	.4848	.9747							
5	200	25	.1449	.5320	.9980							

FIGURE 4. Graph of P(Y $\geq$ y*) for Airdrop using C-130B with y* = 125 items

TABLE 5

Values of $P(Y \geq y^*)$ for Airdrop Using C-130B with $y^* = 50$ Items

k	n	x*	p									
			.05	.1	.2	.3	.4	.5	.6	.7	.8	.9
1,000	1	1	.0500	.1	.2	.3	.4	.5	.6	.7	.8	.9
500	2	1	.0975	.19	.36	.51	.64	.75	.84	.91	.96	.99
250	4	1	.1855	.3439	.5904	.7599	.8704	.9375	.9744	.9919	.9984	1.0000
200	5	1	.2262	.4095	.6725	.8319	.9222	.9688	.9897	.9975	1.0000	
125	8	1	.3366	.5695	.8322	.9424	.9832	.9961	.9993	1.0000		
100	10	1	.4013	.6513	.8926	.9718	.9940	.9990	1.0000			
50	20	2	.6415	.8784	.9885	.9992	1.0000					
40	25	2	.3576	.7288	.9726	.9984						
25	40	2	.6009	.9195	.9985							
20	50	3	.4595	.8883	1.0000							
10	100	5	.5640	.9763								
5	200	10	.5453	.9965								

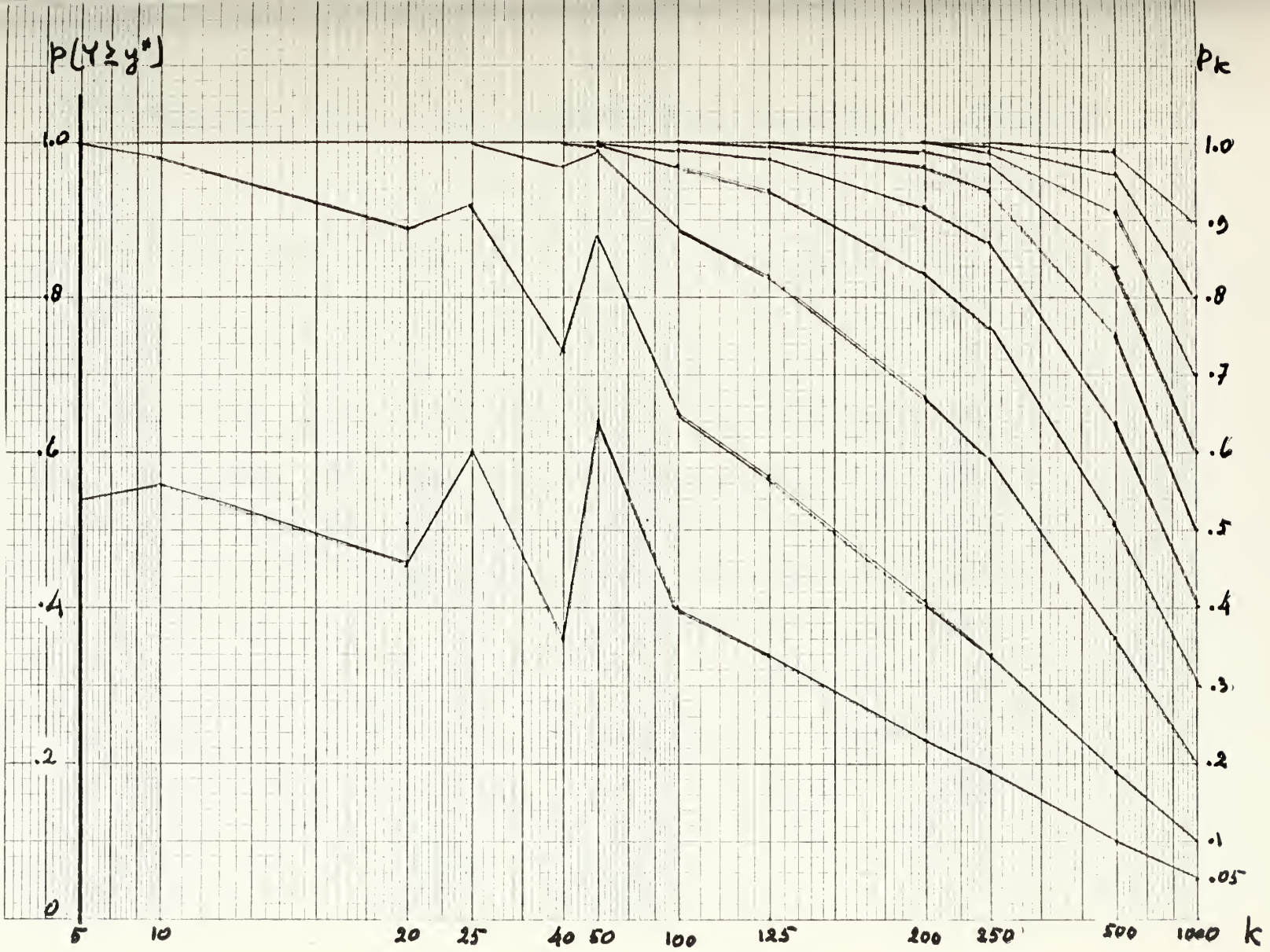


FIGURE 5. Graph of $P(Y \geq y^*)$ for Airdrop using C-130B with $y^* = 50$ items

TABLE 6

Values of $P(Y \geq y^*)$ for Airdrop Using C-47 with $y^* = 200$ Items

k	n	x*	p								
			.1	.2	.3	.4	.5	.6	.7	.8	.9
200	1	1	.1	.2	.3	.4	.5	.6	.7	.8	.9
100	2	1	.01	.04	.09	.16	.25	.36	.49	.64	.81
50	4	3	.001	.0016	.0081	.0256	.0625	.1296	.2401	.4096	.6561
40	5	4	0.0000	.0003	.0024	.0102	.0313	.0778	.1681	.3277	.5905
25	8	7		0.0000	.0001	.0007	.0039	.0168	.0576	.1678	.4305
20	10	9			0.0000	.0001	.0010	.0060	.0282	.1074	.3487
10	20	20						0.0000	.0008	.0115	.1216
8	25	25							0.0000	.0038	.0148
5	40	40								0.0000	.0052
4	50	50									.0002
2	100	100									
1	200	200									

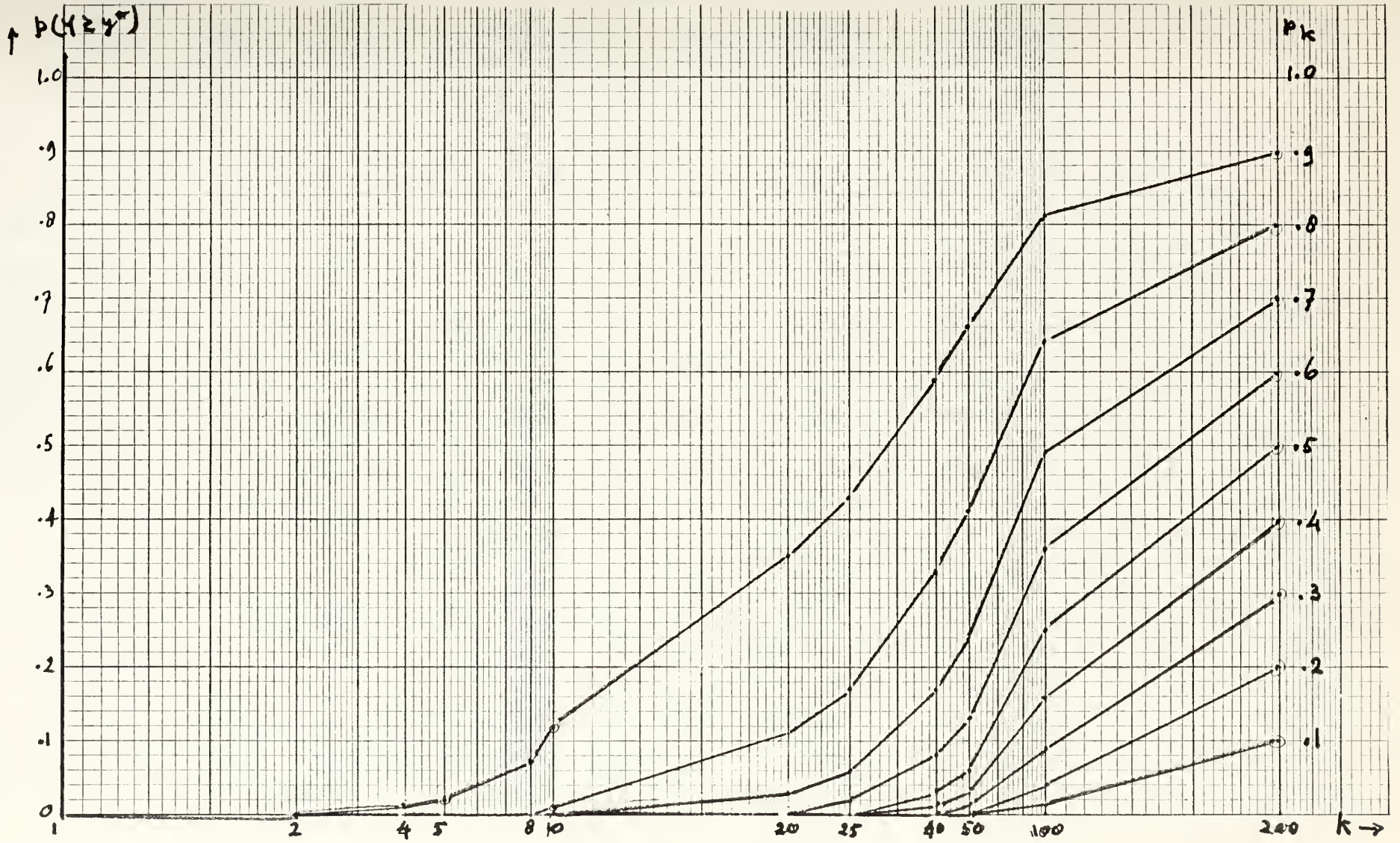


FIGURE 6. Graph of $P(Y \geq y^*)$ for Airdrop using C-47 with $y^* = 200$ items

TABLE 7

Values of $P(Y \geq y^*)$ for Airdrop Using C-47 for $y^* = 100$ Items

k	n	x*	p								
			.1	.2	.3	.4	.5	.6	.7	.8	.9
200	1	1	.1	.2	.3	.4	.5	.6	.7	.8	.9
100	2	1	.19	.36	.51	.64	.75	.84	.91	.96	.99
50	4	2	.0523	.1808	.3483	.5248	.6875	.8208	.9163	.9728	.9963
40	5	3	.0086	.0579	.1631	.3174	.5000	.6826	.8369	.9421	.9914
25	8	4	.0050	.0563	.1941	.4059	.6367	.8263	.9420	.9896	.9996
20	10	5	.0016	.0328	.1503	.3669	.6230	.8338	.9527	.9936	1.0000
10	20	10	0.0000	.0026	.0480	.2447	.5881	.8725	.9829	.9994	
8	25	13		.0004	.0174	.1538	.5000	.8462	.9825	1.0000	
5	40	20		0.0000	.0063	.1298	.5627	.9256	.9976		
4	50	25			.0024	.0978	.5561	.9427	.9991		
2	100	50			0.0000	.0271	.5398	.9832			
1	200	100				.0026	.5282	.9983			

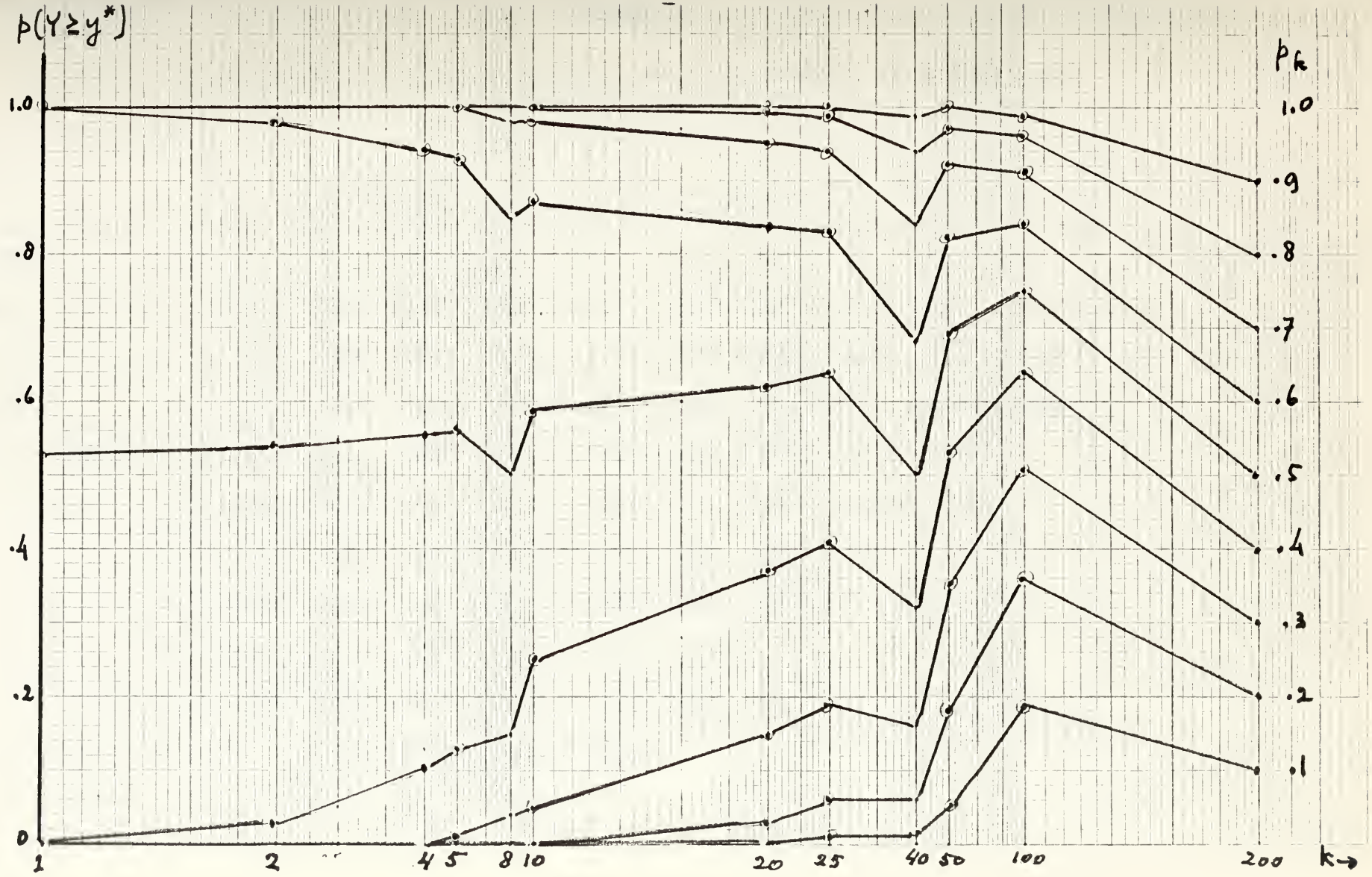


FIGURE 7. Graph of $P(Y \geq y^*)$ for Airdrop using C-47 with $y^* = 100$ items

TABLE 8

Values of $P(Y \geq y^*)$ for Airdrop Using C-47 with $y^* = 50$ Items

k	n	x*	p									
			.1	.2	.25	.3	.4	.5	.6	.7	.8	.9
200	1	1	.1	.2	.25	.3	.4	.5	.6	.7	.8	.9
100	2	1	.19	.36	.4375	.51	.64	.75	.84	.91	.96	.99
50	4	1	.3439	.5904	.6836	.7599	.8704	.9375	.9744	.9919	.9984	.9999
40	5	2	.0815	.2627	.3672	.4718	.6630	.8125	.9130	.9692	.9932	1.0000
25	8	2	.1869	.4967	.6329	.7447	.8936	.9648	.9915	.9987	.9999	
20	10	3	.0702	.3222	.4744	.6172	.8327	.9453	.9877	.9984	1.0000	
10	20	5	.0431	.3704	.5852	.7625	.9490	.9941	.9997			
8	25	7	.0095	.2200	.4389	.6593	.9264	.9926	1.0000			
5	40	10	.0051	.2682	.5605	.8041	.9844	.9997				
4	50	13	.0010	.1860	.4890	.7771	.9868	1.0000				
2	100	25		.1314	.5383	.8864	1.0000					
1	200	50		.0494	.5271	.9494						

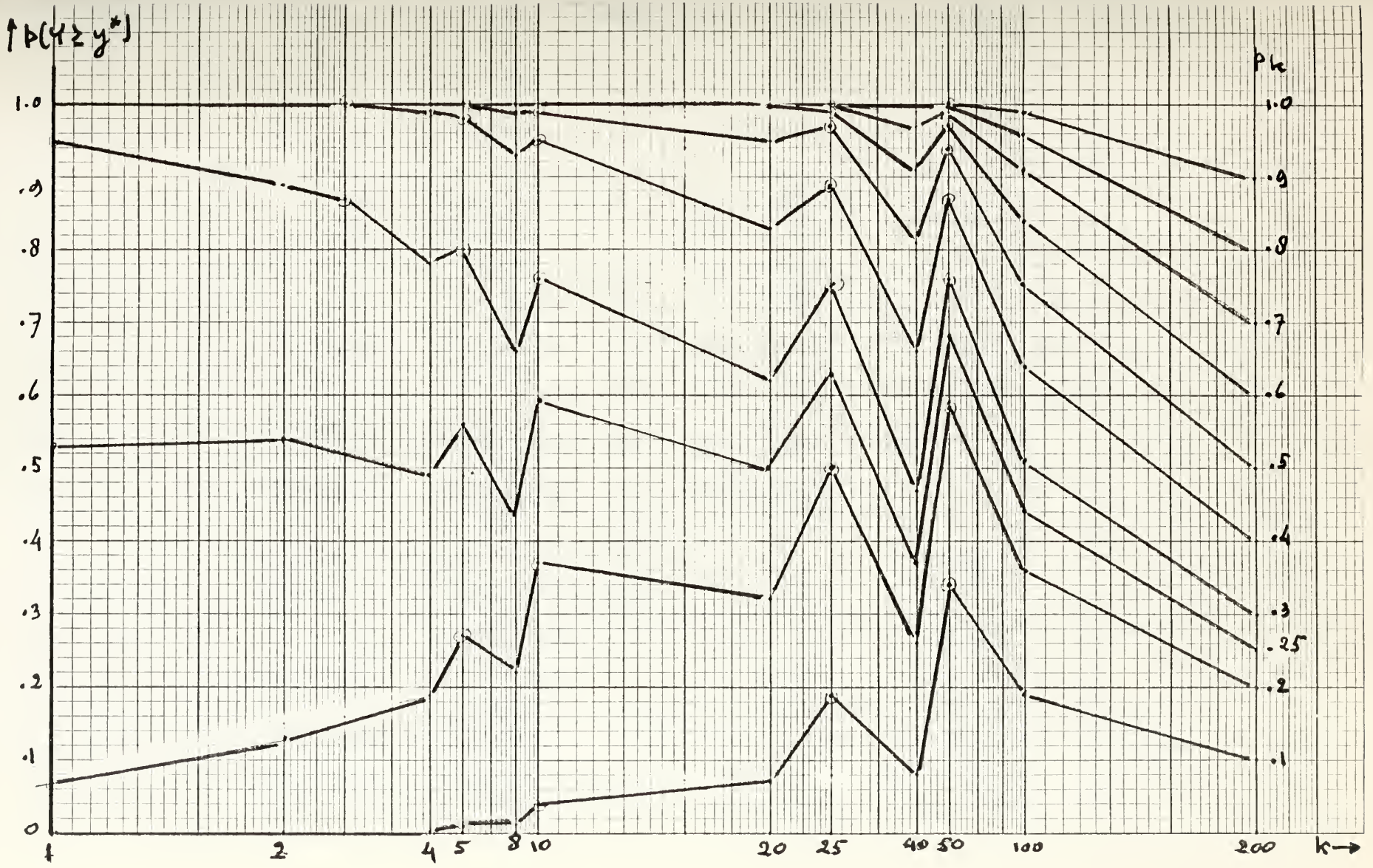


FIGURE 8. Graph of $P(Y \geq y^*)$ for Airdrop using C-47 with $y^* = 50$ items

TABLE 9

Values of $P(Y \geq y^*)$ for Airdrop Using C-47 with $y^* = 25$ Items

k	n	x*	p									
			.1	.125	.2	.3	.4	.5	.6	.7	.8	.9
200	1	1	.1	.1250	.2	.3	.4	.5	.6	.7	.8	.9
100	2	1	.19	.2344	.36	.51	.64	.75	.84	.91	.96	.99
50	4	1	.3439	.4138	.5904	.7599	.8704	.9375	.9744	.9919	.9984	.9999
40	5	1	.4095	.4871	.6723	.8319	.9222	.9688	.9897	.9975	1.0000	1.0000
25	8	1	.5695	.6564	.8322	.9424	.9832	.9961	.9993	1.0000		
20	10	2	.2639	.3611	.6242	.8507	.9536	.9893	1.0000			
10	20	3	.3231	.4647	.7939	.9645	.9964	1.0000				
8	25	4	.2364	.3824	.7660	.9668	1.0000					
5	40	5	.3710	.5713	.9241	.9974						
4	50	7	.2298	.4363	.8966	.9975						
2	100	13	.1982	.4848	.9747							
1	200	25	.1449	.5320	.9980							

FIGURE 9. Graph of P(Y $\geq$ y*) for Airdrop using C-47 with y* = 25 items

TABLE 10

Values of $P(Y \geq y^*)$ for Airdrop Using C-47 with $y^* = 10$ Items

k	n	x*	p									
			.05	.1	.2	.3	.4	.5	.6	.7	.8	.9
200	1	1	.0500	.1	.2	.3	.4	.5	.6	.7	.8	.9
100	2	1	.0975	.19	.36	.51	.64	.75	.84	.91	.96	.99
50	4	1	.1855	.3439	.5904	.7599	.8704	.9375	.9744	.9919	.9984	1.0000
40	5	1	.2262	.4095	.6723	.8319	.9222	.9688	.9897	.9975	1.0000	
25	8	1	.3366	.5695	.8322	.9424	.9832	.9961	.9993	1.0000		
20	10	1	.4013	.6513	.8926	.9718	.9940	.9990	1.0000			
10	20	1	.6415	.8784	.9885	.9992	1.0000					
8	25	2	.3576	.7288	.9726	.9984						
5	40	2	.6009	.9195	.9985	1.0000						
4	50	3	.4595	.8883	.9987							
2	100	5	.5640	.9763								
1	200	10	.5453	.9965								

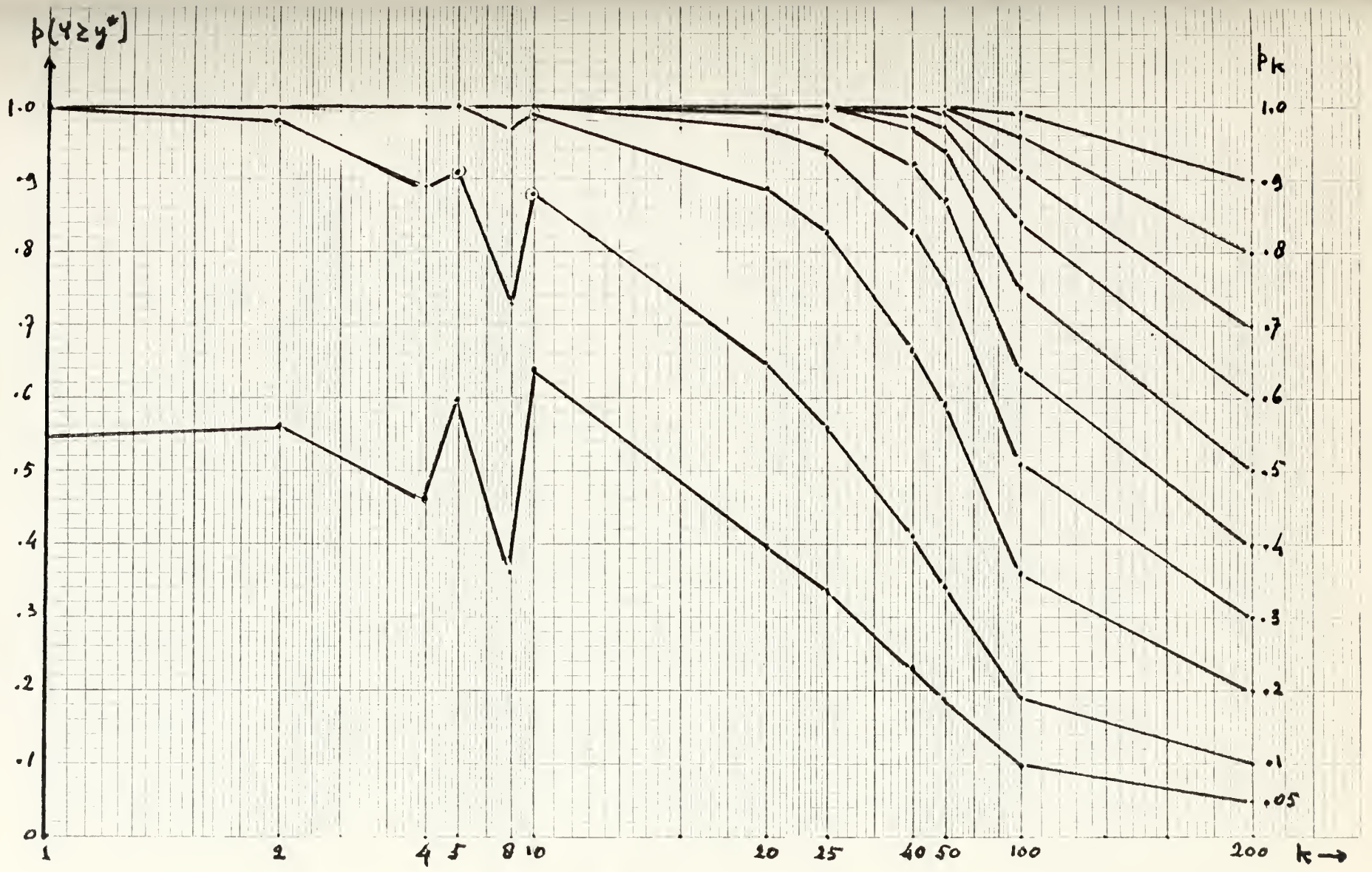


FIGURE 10. Graph of P(Y $\geq$ y*) for Airdrop using C-47 with y* = 10 items

III. APPLICATION TO AIRDROP

A. GENERAL DISCUSSION

1. Description Of The Supply Problem In Indonesia

An airdrop mission is chosen when there is a state of emergency such as earthquake, flood, or the need to support troops in an isolated forward area.

Considering the geographical situation of Indonesia which consists of many islands and spans more than 3000 miles from west to east and more than 1500 miles from south to north, air delivery becomes very important among the islands. For the military, air delivery is a necessity, particularly when speed is very badly needed in supporting our troops if military operations occur on one of the islands. In some of the islands the condition of the roads are either very poor or no roads exist. In Kalimantan (Borneo), transportation is carried out through rivers by boats or barges which will take days to cover only 50 miles. Because of this air delivery becomes important in Indonesia.

2. A Survey Of Tables 1-10 And Figures 1-10

Tables 1 to 10 show values of $P(Y \geq y^*)$ for various values of p_k and k. Tables 1 to 5 are matched with C-130B aircraft and Tables 6 to 10 are matched with C-47 aircraft. Figures 1 to 10 show the graphs of the values of $P(Y \geq y^*)$ in Tables 1 to 10, where Table 1 matched with the graph in Figure 1, Table 2 with Figure 2 an so on. The

graphs are constructed on semi-logarithmic graph paper with the values of $P(Y \geq y^*)$ as ordinates and the values of k on the abscissa.

Now, let $p^* = y^*/m$, so that for the package, survival probability equals p^*, the expected number of surviving packaged items is $\mu^* = mp^* = y^*$. Because we don't know the dependence of p_k on k, it is assumed for the present that p_k has the same value of p for all package sizes.

Let's now examine the values of $P(Y \geq y^*)$ in Tables 1 to 10 and the graphs in Figures 1 to 10. In Tables 1 and 6 and subsequently Figures 1 and 6, where $p^* = y^*/m = 1$, we see that at every p_k, reducing the package size (k) will also reduce the values of $P(Y \geq y^*)$. Examining further Table 2 and Figure 2, where $p^* = 500/1000 = 0.5$, for $p_k > 0.5$, reducing the package size (k) will result in increasing the values of $P(Y \geq y^*)$. On the other hand, for $p_k < p^* = 0.5$, when we reduce the package size (k), first the value of $P(Y \geq y^*)$ will rise until it reaches the highest value at $k = y^* = 500$ and then it will drop down until it goes to zero. If $p_k = p^* = 0.5$, reducing the package size will increase $P(Y \geq y^*)$ until it reaches the highest value at $k = y^*$ (in this case $k = 500$) and then it will reduce to approximately 0.5. If k is not the multiple of y^*, $P(Y \geq y^*)$ will drop, making valleys in the graphs of $P(Y \geq y^*)$. Those properties

of the effects of changing the package size on the package survival probability p_k discussed above, can also be seen in the remaining tables and figures. We come to the conclusion (see Reference 1) that:

a. If $p_k > p^*$, reducing the package size k will result in increasing $P(Y \geq y^*)$.

b. If $p_k = p^*$, reducing k will result in first $P(Y \geq y^*)$ rising until it reaches the highest value at $k = y^*$, and then falling to approximately 0.5.

c. If $p_k < p^*$, reducing k will result in first $P(Y \geq y^*)$ rising until it reaches the highest value at $k = y^*$, and then falling to zero.

3. Two Packaging Problems

a. Type of Aircraft

In this case, an aircraft is considered as a package and the survival probability of the package p_k is the survival probability of the aircraft. We don't know the dependence of p_k on k, but suppose we assume that p_k has the same value p for all package sizes, in this case for all types of aircraft, whether it is C-130 or C-47 or other smaller aircrafts and helicopters. The survival probability of the aircraft depends on many things, including aircraft vulnerability to enemy fire and package size, which will be discussed in the following paragraph.

b. Packaging for Airdrop from an Aircraft

In the case of airdrop from an aircraft, the aircraft acts as a conveyer. The containers carried by the a/c where the items are packaged are considered the packages. The size of the package (k) means how many items are packed in that container. The package survival probability is the probability of survival of the container when dropped from the aircraft to the ground, not the aircraft survival probability we discussed previously. The survival probability of the package also depends on many factors, e.g.,

- Accuracy in hitting the dropzone or targets,
- Methods of dropping,
- Ground effects, and
- Package size.

These factors will be discussed in the following section. The overlapping of the two problems will be discussed in Subsection D.

B. CHOICE OF AIRCRAFT

1. Factors

As already mentioned in Section A above, the package survival probability depends on many factors. The factors which will affect the choice of aircraft are:

- Package Size
- Aircraft vulnerability

We'll discuss these factors in the following paragraphs.

a. Package Size

Using large aircraft to bring all the items in could be very dangerous. On the other hand using many smaller aircraft to bring a smaller portion of the m items could involve considerable cost. Sometimes we are faced with a decision: Should we use one C-130B or five C-47's? As has been discussed in the previous section, for theoretical purposes we considered p_k to have the same value p for all package sizes. In real life, it is unreasonable to expect that all sizes of aircraft have the same survival probability, i.e., that p_k does not change with k. Let's take, for example, the C-130B. Because of its speed and its ability to fly at high altitude, it might have a greater survival probability p_k than a C-47 which is very slow and usually flies at low altitude. The C-47, in turn, might have greater survival probability than a helicopter, which is still slower. Smaller aircraft, although slow, can fly at very low altitude (almost hugging the ground) and are more maneuverable to avoid enemy fire. So, small aircraft, e.g., the Army L-20, might have greater survival probability compared to a helicopter or even to a C-47. If at least the desired number of items y^* are to survive, Table 11 shows how the changes in the package size (in this case the type of aircraft) could change the value of $P(Y \geq y^*)$. Each type of aircraft has a different survival probability p_k, and it varies from one dropzone to the

other. In Table 11, the value of p_k for each type of a/c has been taken arbitrarily.

TABLE 11

PROBABILITY OF AT LEAST Y* = m/2 ITEMS SURVIVING BY USING DIFFERENT TYPES OF AIRCRAFT

Type of Aircraft	k	n_k	p_k	$P(Y \geq y^*) = P(X \geq x)$
C-130B	m	1	0.7	$P(X = 1) = 0.7$
C-47	m/5	5	0.6	$P(X \geq 3) = 0.6026$
Helicopter	m/10	10	0.5	$P(X \geq 5) = 0.6231$
Smaller Helicopter	m/20	20	0.5	$P(X \geq 10) = 0.5881$
Small Aircraft	m/50	50	0.55	$P(X \geq 25) = 0.8034$

From Table 11 it is seen that the probability that at least $y^* = m/2$ survive, varies with the type of aircraft being used and we get a clearer picture as to what aircraft will be used. Employing many small aircraft will give a higher $P(Y \geq y^*)$, but a small aircraft has other shortcomings which will be discussed later.

b. Aircraft Vulnerability

Using relatively slow moving aircraft in forward areas, at least partially occupied by enemy elements, will increase the vulnerability of the aircraft to enemy fire. It is evident that the survival probability of the aircraft

is an important consideration. Besides costs and highly skilled crews, the continuation of operations is a factor because of aircraft inavailability if many are lost (Indonesia has very limited aircraft).

As an example, suppose the survival probability of the C-47 (p_k) is 0.4 , thus the prbability of neutralizing it is 1 - 0.4 = 0.6 . In Table 12 we can see the effects of changes in the number of aircraft used, on the probability of at least 3/4 of the aircraft (packages) surviving. In this table we use the C-47 as a package with $p_k = 0.4$ and

z = number of items on surviving aircraft

w = number of packages (aircraft) surviving.

TABLE 12

EFFECTS OF THE NUMBER OF THE AIRCRAFT USED ON THE PROBABILITY OF AT LEAST 3/4 OF THE AIRCRAFT SURVIVING OR AT MOST 1/4 OF THE ITEMS ARE LOST

Number of C-47 used (n_k)	$P(z \geq 3/4\ m)$ or $P(W \geq w)$
1	$P(W = 1) = 0.4$
4	$P(W \geq 3) = 0.179 \simeq \frac{1}{6}$
6	$P(W \geq 4) = 0.041 \simeq \frac{1}{25}$
20	$P(W \geq 15) = 0.002 \simeq \frac{1}{500}$

From Table 12 it is obvious that increasing the number of aircraft used will decrease the probability that at least 3/4 of the aircraft survive, hence 3/4 of the items survive (see page 38, where $p_k = 0.4 < p^* = 0.75$).

If four aircraft are used, the probability that at least 3/4 of the items surviving (or 3/4 of the aircraft surviving) is:

$$P(W \geq 3) = 0.179 \approx \frac{1}{6},$$

and we can say that one out of 6 missions or sorties can be expected to lead at most to neutralization of 1/4 of the aircraft, hence 1/4 of the items. Thus if one aircraft carries 200 items, then the chance to lose at most 50 items is 0.179 (See Table 12). If 6 aircraft are used, then the probability that at least 3/4 of the aircraft survive is $0.041 \simeq \frac{1}{25}$, one out of 25 missions will lead to neutralization of at most 1/4 of the aircraft or items (3/4 of 6 aircraft is 4.5, but the number of aircraft must be an integer, so it's rounded down to 4). Increased again to 20 aircraft, one out of 500 missions will be needed to neurtralize at most 1/4 of the items or 1/4 of the aircraft. The more aircraft employed, the smaller the probability of the survival of at least 3/4 of them. From Table 12 and the discussion in the previous paragraph, we get a clearer picture in making a decision, i.e., which type of aircraft to employ and how many to use.

2. Discussion

As has been illustrated in the computations above, using different package sizes (smaller aircraft) can vary the probability of at least the desired number of items surviving. If $p_k > p^*$, employing many small aircraft will give a higher $P(Y \geq y^*)$, but a small aircraft has other shortcomings such as short range, small payload, etc. As an example, instead of using one C-130 B, five C-47's can be used all at once to the dropzone or one C-47 five times. But using one C-47 five times to the dropzone might take too much time to complete the desired number of items. This may not be acceptable for military operations where every second counts, and the case of repeated use of one aircraft during an operation will not be considered here.

The package survival probability p_k will decrease substantially if air supremacy is in the hands of the enemy. Evasive action and selection of the safest flight pattern should be employed to increase p_k. The long-range aircraft are necessary in Indonesia because of the geographical situation which has been discussed on page 36.

C. PACKAGING PROBLEMS IN AIRDROP

1. Factors

In discussing airdrops, the aircraft is considered as a conveyer only and the container as the package. An aircraft can carry more than one package, the number depending on the capacity of the aircraft (m) and on the

number of items in a package (k) . An airdrop can fail, not only because of the packages being damaged when hitting the ground, but also because of the possibility of loss if items are not recovered, particularly if the dropzone is a jungle area. Besides the techniques of packaging which will not be discussed here, theere are many factors which affect the package survival probability, some of them are:

- Accuracy of drop
- Package size
- Method of drop
- Ground effects.

These factors will be discussed one by one in the following paragraphs.

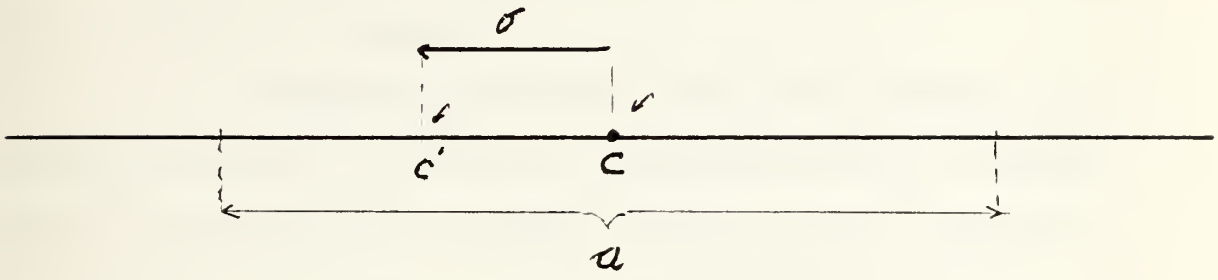
a. Accuracy of Drop

There is a requirement for airdrops that the drop zone must be easily located by the aircraft under all conditions of terrain and weather, in daylight or at night. However there are many other elements that affect the package hitting the ground accurately. Wind, for example, can push the package outside the dropzone where it could fall into the enemy's hands.

Let's compute the probability of the accuracy of hitting the dropzone. First, we consider a one dimensional dropzone. Suppose an aircraft drops the package on a drop-zone with length a and let the impact point be a random variable X normally distributed on that line (see Fig 11).

Figure 11

One Dimensional Dropzone



Assuming the distribution of the pont where the package hits the ground is centered at C with mean $\mu = 0$, and, standard deviation σ, then the density distribution of x is:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2} , \quad \text{(See Ref. 6)}$$

and the probability of a hit on the dropzone centered at C is:

$$\int_{-a/2}^{a/2} f(x)\, dx .$$

The probability of hit on the dropzone will be:

$$\Pr(\text{hit}) = \Pr(-a/2 \leq X \leq a/2) = \Phi(a/2) - \Phi(-a/2\sigma)$$

$$= 2\Phi(a/2\sigma) - 1 ,$$

where $\Phi(x) = Pr(X \leq x)$ can be obtained from standard normal tables. The standard deviation can be estimated or computed using analytical and experimental methods.

Before we go on into the two dimensional case, let's look at an example:

Suppose an aircraft drops its package on a line with length $a = 100$ m . Assuming that the impact points centered in the middle of the line are normally distributed with mean $\mu = 0$ and standard deviation $\sigma = 38$ m , then the probability of a hit is:

$$Pr\ (hit) = 2\Phi(50/38) - 1 = 2\Phi(1.3158) - 1$$

$$= 2(0.9059) - 1 = 0.8118 .$$

In two dimensional cases, we work with two random variables X and Y . It is assumed that X and Y are independently and normally distributed $N(X; 0; \sigma_x^2)$, $N(Y; 0, \sigma_y^2)$ and $\sigma_x^2 = \sigma_y^2 = \sigma^2$. The density functions of X and Y are:

$$f_X(x) = \frac{1}{\sigma_x \sqrt{2\pi}} e^{-x^2/2\sigma_x^2}$$

and

$$f_Y(y) = \frac{1}{\sigma_y \sqrt{2\pi}} e^{-y^2/2\sigma_y^2} .$$

The joint density is

$$f_{X,Y}(x,y) = f_X(x) \cdot f_Y(y)$$

$$= \left(\frac{1}{\sigma_x \sqrt{2\pi}} e^{-x^2/2\sigma_x^2}\right)\left(\frac{1}{\sigma_y \sqrt{2\pi}} e^{-y^2/2\sigma_y^2}\right) .$$

If it is assumed that $\sigma_x = \sigma_y = \sigma$, then:

$$f_{X,Y}(x,y) = \frac{1}{2\pi\sigma^2} e^{\frac{X^2+Y^2}{2\sigma^2}} .$$

The probability of hit in area A is:

$$\Pr(\text{hit}) = \int\int_A \frac{1}{2\pi\sigma^2} e^{-\frac{X^2+Y^2}{2\sigma^2}} \, dx \, dy .$$

If we assume that the dropzone is circular with radius R , we now transform to polar coordinates as follows:

$$X = r \cos\theta , \quad Y = r \sin\theta ,$$

and thus

$$X^2 + Y^2 = r^2 \cos^2\theta + r^2 \sin^2\theta$$

$$= r^2(\cos^2\theta + \sin^2\theta)$$

$$= r^2 .$$

Also,

$$dx \cdot dy = r \cdot dr \cdot d\theta .$$

The probability of a hit will be:

$$\Pr(\text{hit}) = \frac{1}{2\pi\sigma^2} \int_0^R \int_{-\pi}^{\pi} r\, e^{-r^2/2\sigma^2}\, dr\, d\theta$$

$$= \frac{2\pi}{2\pi\sigma^2} \int_0^R r\, e^{-r^2/2\sigma^2}\, dr$$

$$= -e^{-r^2/2\sigma^2} \Big]_0^R$$

$$= 1 - e^{-R^2/2\sigma^2} .$$

Example:

If the dropzone is circular with diameter $a = 100$ m , and the standard deviation is $\sigma = 38$ m , then:

$$PR\ (hit) = 1 - e^{-R^2/2\sigma^2}$$

$$= 1 - e^{-50^2/(2)(38)^2}$$

$$= 1 - e^{-0.8657}$$

$$= 0.5792 \quad .$$

It is obvious that if R is increased, then the probability of hitting the dropzone will also increase. On the other hand, if the standard deviation increases, the probability of hitting the dropzone decreases.

b. Package Size

Suppose our troops in forward areas need 500 items of supplies, and we use a C-130 B aircraft for dropping them. Then we have $m = 1000$ and $y^* = 500$ (see Table 2 and Figure 2). If the package survival probability is estimated at 0.4 , then when the items are packed in one package, the package has theprobability 0.4 to survive. If the package size is reduced to $k = 500$ items, then there are 2 packages of size 500 each. The probability that at least $y^* = 500$ items survive (or in other words

one out of the two packages will survive) will be:

$$P(Y \geq y^*) = P(X \geq 1) = 0.64 .$$

The probability of the desired number of items to survive is increasing. If the package size is reduced again to $k = 250$, the probability that at least $y^* = 500$ items (2 out of 4 packages) survive will be:

$$P(Y \geq y^*) = 0.5248 .$$

The probability at least $y^* = 500$ survive is decreasing at $k = 250$ and will go on decreasing if k is decreased further. In this case $p^* = 0.5$ and $p = 0.5$, and $P(Y \geq y^*)$ is the highest at $k = 500$ (see page 38).

If $p = 0.7$, reducing package size will increase $P(Y \geq y^*)$ and at $k = 10$, $P(Y \geq y^*) = 1$. From these computations, it is obvious that the package size plays an important role to increase the probability of the desired number ($P(Y \geq y^*)$ surviving.

c. Method of Drop

Before the airdrop is carried out, we have to decide which method is going to be used, i.e., which is suitable with the conditions of the dropzone so as to increase the package survival probability. There are three methods of airdrop:

- the retarded fall method,
- the high velocity method, and
- the freefall method.

These are discussed one-by-one as follows:

(1) Retarded Fall Method

The retarded fall method employs parachutes to reduce the rate of descent of the items dropped. It is obvious that the retarded method gives a higher survival probability against damage from impact with the ground, but it also has less accuracy in hitting the dropzone. This method is not suitable if the dropzone is in a jungle area.

(2) High Velocity Method

The high velocity method of dropping employs pilot parachutes to orient and stabilize the falling package. It is more accurate in hitting the dropzone than the retarded fall method, but gives a higher probability of damage to the package and its contents. Whether it has a higher or lower package survival probability depends on the size of the dropzone and the susceptability of the packaged items to damage. When the first method has unacceptably low package survival probability because of restrictions in dropzone, the second method provides the possibility of increasing p_k where additional protection such as cushioning may be needed to reduce damage at impact. Note that the use of additional packaging material may also reduce the number of items that can be carried in the aircraft (smaller m).

(3) Free-fall Method

The free-fall method of delivery simply involves dropping packages from aircraft to the ground, and was used at times in World War II in various theaters, particularly in the Pacific and Asia. The advantages of the free-fall method compared to descent by parachute are:

- It has a greater speed reaching the ground,
- It has a greater accuracy in hitting the dropzone, and
- Because no parachute is needed in using this method, it is less expensive.

The disadvantage is the greater susceptability of items to damage on impact with the ground.

Each method has its own advantages. Sometimes the cost is taken into consideration to decide which method is the cheapest and the safest. As an example, if we are short of parachutes and the dropzone is a jungle area, the free-fall method seems to be the right method and is usually the cheapest, particularly for Indonesia. With other methods, i.e., retarded or high velocity methods, usually the chutes may be caught in high trees and it is very difficult to get them down. Sometimes in the jungle dropzone, nature helps lower the damage caused by the free-fall because the packages get some chushioning by ground vegetation on hitting the ground. This increases the survival probability of the packages. The free-fall method can

also be used for a small dropzone, since it need greater accuracy in hitting the dropzone and greater speed to reach the ground. Using other methods, there is a high probability the packages will be lost; either they are difficult to recover or they are damaged by enemy action. In Indonesia where the trouble spots are usually jungle areas, free-fall method with a small-sized packaged items is usually recommended. "The supplies dropped must be ruggedly packaged, inherently indestructable, or both, to preclude damage." Figure 12 shows the spectrum of air delivery methods, in relation to vulnerability to enemy fire, damage to supplies and cost delivery equipment (see Reference 3).

d. Ground Effects

Ground effects sometimes can cause a considerable damage or failure to the package dropped. Some of them are as follows:

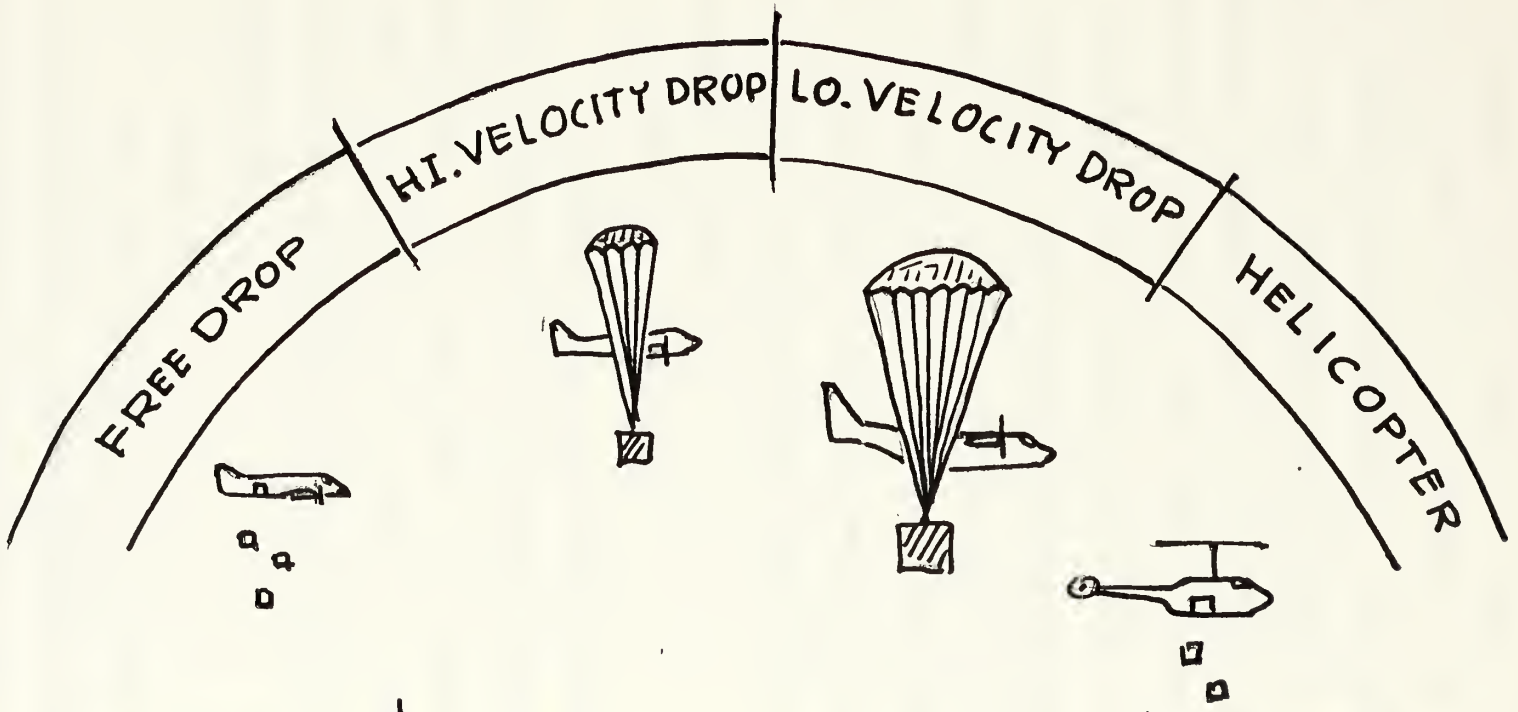
- Condition of the terrain,
- Troop concentration,
- Weather,
- Enemy, and
- Communication.

We'll discuss these briefly in the following paragraphs.

(1) Condition of the Terrain

If the dropzone is in a jungle area, the probability of loss is much higher than in an open field. According to the experience in New Guinea Campaign in

AIR DELIVERY METHODS SPECTRUM



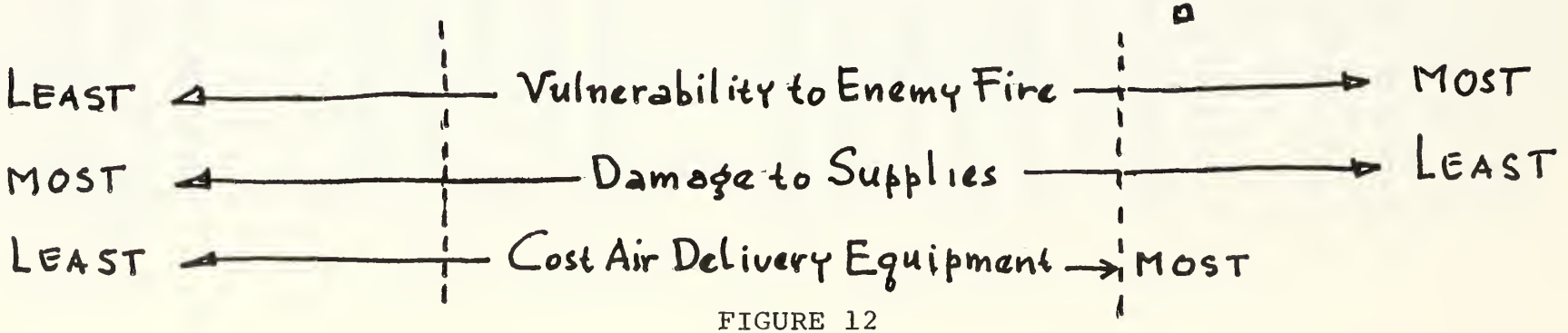


FIGURE 12

WW II (see Ref. 4), the drops were made at a 300-foot altitude and required three or more runs over the dropzone. Losses of the supplies due to force of impact with the ground were only about 5 to 10 percent, but recoveries varied widely according to the terrain. In comparatively open plantation areas, recovery was practically 100 percent, whereas in dense jungles, only about half the supplies were found. Beside that, when parachutes caught in trees, the suspension lines had to be shot off with substantial damage to supplies and containers dropping to the ground and with total loss of the parachutes. In more open areas, 90 percent of chutes and containers were recovered in a servicable condition, but so much time was used to get them back to base through the jungle that high losses were caused by humidity.

(2) Troop Concentration

A bigger force can occupy a larger area, making a larger dropzone so as to increase the package survival probability. In some cases, the dropzone can be far behind the front lines, out of range of enemy fire, to decrease aircraft vulnerability (increasing aircraft survival probability).

(3) Weather

The role of the weather in the dropzone area must not be belittled. Bad weather can stop or hinder airdrop operations because of strong wind, fog, rains etc.

Strong wind can push the packages outside the dropzone, letting them fall into the hands of the enemy. In other words, a strong wind can cause a greater standard deviation of the impact point. The greater the standard deviation, the lower the probability of hitting the dropzone (see page 50).

Fog can cause inability to locate the dropzone accurately and this can result in a low probability of hitting the dropzone.

Rains and thunderstorms can make the dropzone soaked with water and change it into a muddy area. This can cause the dropzone to be difficult to recognize from the air. Sometimes the water will ruin the items in the packages.

(4) Enemy Fire

If the dropzone is not far away from the front line, it can be pinpointed by enemy fire, causing considerable damage to packages dropped and casualties to personnel.

(5) Communications

Communications are very important for a successful airdrop mission. Communications take place from the ground to the aircraft, to guide them to the dropzone in bad weather. Communications also take place among the troops on the ground, to designate dropzones, and to notify homebase on the location of dropzones. Many other things can be done rapidly and accurately by communications so as to increase survival probability.

D. OVERLAP OF THE TWO PACKAGING PROBLEMS

The two packaging problems we have discussed involve:

1. The aircraft as a package, and
2. The aircraft as a conveyer.

More often than not, an aircraft involved in an airdrop plays the role as a package as well as a conveyer. There are two probabilities that are importnat here:

- The survival probability of the aircraft.
- The survival probability of the packages (supplies) carried by the aircraft.

Suppose we want to drop supplies using only one C-130B which has the capacity $m = 1000$. Let the desired number of items surviving be

$$y^* = m/2 = 500 ,$$

and the size of the container be

$$k = m/2 \quad \text{or} \quad n_k = 2 .$$

If it is known that the aircraft survival probability is:

$$P_{C\text{-}130} = 0.6$$

and the container survival probability is

$$p_k = 0.5 ,$$

then the probability $P(Y \geq y^*) = 0.75$ for package survival after it is dropped from the aircraft must be multiplied by the probability of C-130 surviving. Thus:

$$P_{C\text{-}130} \cdot P(Y \geq y^*) = (.6)(.75) = .45$$

is the probability that the desired number of items survive when a single C-130 carrying packages of $k = 500$ items is used. If n_k is the number of packages in the aircraft and J is the number of packages surviving, then using one C-130:

a. $k = m$, $y^* = m/2$, $n_k = 1$, and

$$P(Y \geq y^*) = P(X = 1)\ P(J = 1)$$

$$= P_{C\text{-}130} \cdot P_k$$

$$= (0.6)\ (0.5) = 0.3 .$$

b. If $k = m/2$, $y^* = m/2$, $n_k = 2$, then

$$P(Y \geq y^*) = P(X = 1)\ P(J \geq 1)$$

$$= P_{C\text{-}130}\left[\sum_{m=1}^{2} P(J = j)\right]$$

$$= (0.6)\ (0.75) = 0.45 .$$

c. If $k = m/4$, $y^* = m/2$, $n_k = 4$, then

$$P(Y \geq y^*) = P(X = 1)\ P(J \geq 2)$$

$$= P_{C\text{-}130} \left[\sum_{j=2}^{4} P(J = j) \right]$$

$$= (0.6)\ (0.6075) = 0.4125 .$$

The value for $P(J \geq j)$ came from Table 2.

We would have to use five C-47's to match the capacity of one C-130B. If all five aircraft survive, then survival of at least half the packages on each aircraft will assure the desired item survival level. However, this is not the only way to achieve the desired result. For example, if four of the C-47's had package survivals of 75% , the desired item survival would be satisfied even if none of the packages on the fifth C-47 survived. Further, with $P_{C\text{-}47} = 0.5$ the probability that all the C-47's will survive is $(.5)^5 = 0.03125$. At least 3 of the C-47's must survive in order to have a chance for $m/2$ items to survive. Then we must consider package survival for drops from each of the surviving aircraft.

As an example, let's have a hypothetical aircraft which has a capacity $m/2$ items. We need 2 aircrafts if the size of package on an aircraft is $m/2$, then the number of packages on an aircraft is one. If we need $m/2$ as the desired number (y^*) , the computations are as follows:

capacity = m/2 items and y^* = m/2

z = number of aircrafts used

n_{ac} = number of packages on an aircraft

In this case n_{ac} = 2 and $x^* = 1$. Suppose $P_{ac} = 0.5$ and $P_k = 0.6$. Then

$$P(Y \geq y^*) = P(X \geq x^*)$$

$$= \sum_{z=1}^{2} P(Z = z)\, P(X \geq x^* | Z = z) .$$

$$P(Z = z) = \binom{Z}{3} P_{ac}^{\,3} (1 - P_{ac})^{Z-z}$$

$$P(Z = 1) = \binom{2}{1} (0.5)\,(0.5) = 0.5 .$$

$$P(Z = 2) = \binom{2}{2} (0.5)\,(0.5) = 0.25 .$$

$$P(X \geq x^* | Z = 1) = \sum_{m=x^*}^{n_{ac}} \binom{n_{ac}}{m} p_k^{\,m} (1 - p_k)^{n_{ac}-m}$$

$$= \binom{1}{1} (0.6)^1 (0.4)^0 = 0.6 .$$

$$P(X \geq x^* | Z = 2) = \sum_{m_1=1}^{n_{ac}} \sum_{m_2=1}^{n_{ac}} P(M_1 = m_1)\, P(M_2 = m_2)$$

$$= (0.6)\ (0.6) = 0.36 .$$

Therefore,

$$P(Y \geq y^*) = (0.6)\ (0.5) + (0.36)\ (0.25)$$

$$= 0.39 .$$

Suppose now, each aircraft uses two containers of $m/4$ size, then for the desired number of items to survive, $P(Y \geq m/2)$, there must be 2 packages at least surviving. The computation will be:

4 packages survive from 2 a/c:

$$(.5)^2[(.6)^2][(.6)^2] = 0.0324$$

3 packages survive, 1 from 1 a/c, 2 from the other:

$$(.5)^2[(.6)(.4)(.6)^2 + (.6)^2(.6)(.4)] = 0.0432$$

2 packages survive, either 2 packages from one a/c or 1 package from each a/c :

$$(.5)^2[2(.4)^2(.6)^2 + 2(.6)(.4)(.6)(.4)]$$

$$+ 2(.5)^2[(.6)^2] = 0.2376 .$$

Therefore,

$$P(Y \geq m/2) = P(X \geq 2) = 0.0324 + 0.0432 + 0.2376$$

$$= 0.3132 .$$

From the computation we conclude that using one package on each of the two aircraft $(k = m/2)$ yields a higher value for $P(Y \geq m/2)$ than using two packages on each of the aircraft $(k = m/4)$. Note that the probability of package survival (p_k) was the same for both package sizes. If p_k were larger for the smaller package, the conclusion would not necessarily be the same.

Using one C-130 with package size $k = m/2$ we have $P(Y \geq m/2) = .45$. Using two hypothetical aircraft with capacity half that of the C-130 and $k = m/2$ we find $P(Y \geq m/2) = 0.39$. Clearly the C-130 is preferable. Note that this involves the probabilities of aircraft and package survival for the two aircraft sizes and hence the conclusion could be different if these are changed.

IV. CONCLUSIONS AND RECOMMENDATION

The applications of the Binomial model discussed in Section II, particularly concerning airdrop, must be developed further for application in Indonesia. The applications to airdrop developed in Section III are mostly based on assumptions because of lack of real data.

The aircraft used in the applications to airdrop and all the examples are C-47 and C-130B. C-47's in most developed countries have been considered as obsolete, and in a few more years will also be obsolete in Indonesia. So experimenting about packaging with newer and more modern aircraft is still a challenge in Indonesia.

In the discussion about the accuracy of drop, it was shown that probability of hitting the dropzone depends on the diameter of the dropzone (assumed circular form) and also on the standard deviation of hits. The distribution of hitting points was assumed normal. This assumption has to be examined by experimentation. The estimate on standard deviation must be studied experimentally and analytically.

Cost considerations have not been discussed, but it is obvious that air delivery is costly, as far as the aircraft are concerned. In a country like Indonesia, however, it is hard to avoid air delivery. We have enough packaging materials for airdrops, but the parachutes are still expensive.

An even greater cost factor, however, is that incurred by materials being lost in airdrops and to have them

recovered by enemy forces. New decision models are needed to balance what may be an exponential cost to friendly troops who are already low on food and ammunition who suddenly find their enemy resupplied.

Such new models should include correlates of visibility and topographical factors. The likelihood of ground fog and monsoons has been estimated. Areas have been mapped showing double or triple canopy. The final equation for the military decision maker should include these factors.

The model and its applications described in this thesis need more development and experimentation, particularly the survival probability of the packages dropped in various types of dropzones and at various situations and conditions (jungle areas, not adequate ground facilities, etc.).

The author hopes that further experimentations will be useful for obtaining higher package survival probability to airdrop in the Indonesian Armed Forces.

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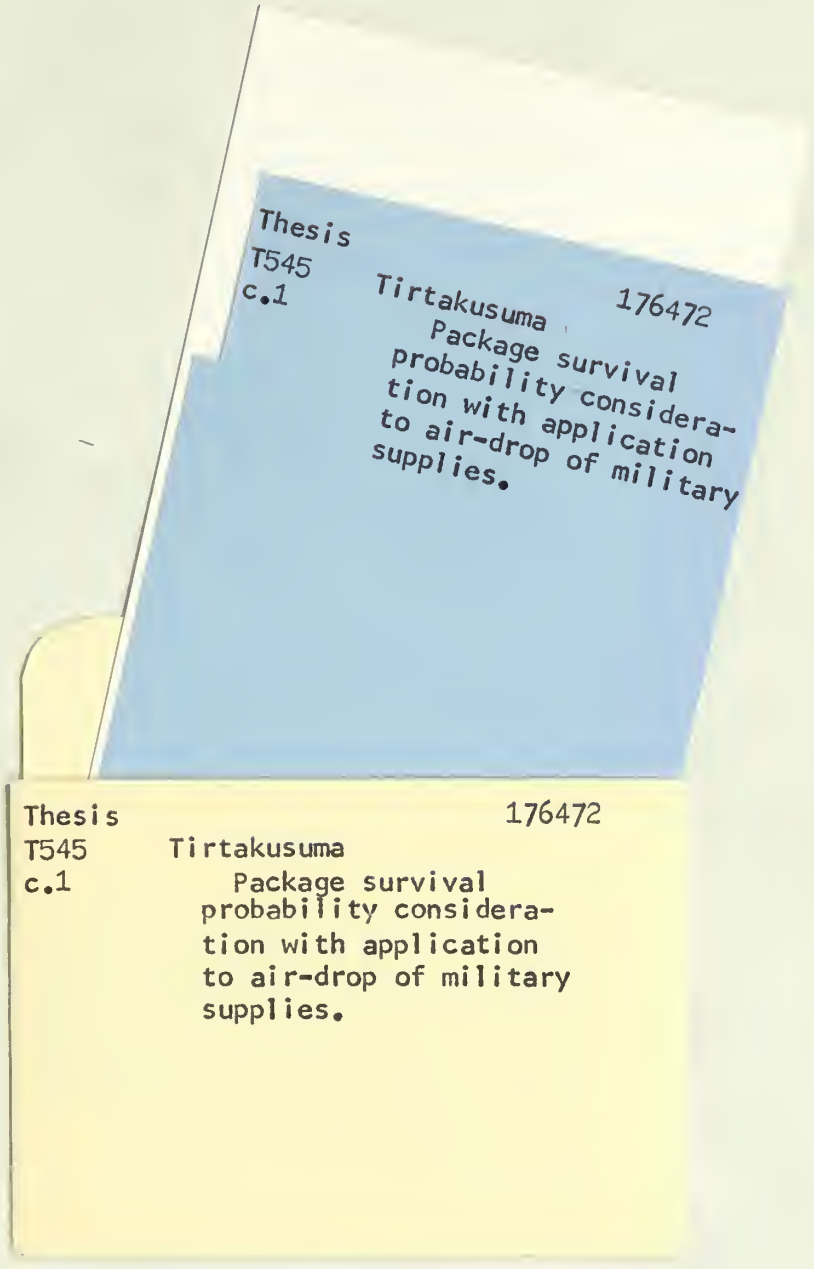

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